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COMPLEX MODEL OF THE EFFICIENCY OF RECTIFICATION PLATES. 5. CROSS MOTION OF THE PHASES IN MIXING OF A LIQUID

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Partial mixing of a liquid in cross motion of the phases is considered. Part of the liquid (φ) on a plate is totally mixed, while the other part (1 – φ) moves in the regime of ideal displacement; the degree of mixing is determined by the relation of these parts. The composition of a vapor after the plate is taken to be equal to the average value of the compositions of the vapor that come out of the initial and final portions of the plate in the direction of liquid motion. After an ideal plate these compositions are in equilibrium with the arriving and leaving liquid, respectively. The analytical dependences of the efficiency on the parameters of a real plate are obtained. The equality of the efficiencies in the vapor and liquid phases is proved. Certain particular and limiting cases of applicability of the complex model are considered.

It is noted in [1] that in countercurrent motion of the phases the mixing of a liquid has an appreciable effect on the efficiency of mass transfer, decreasing it as compared to ideal displacement. A special feature of the mixing in countercurrent flow is its more substantial effect in the direction of the liquid flow and the less appreciable effect in the perpendicular direction. The degree of mixing of the liquid is evaluated by the relation between its totally mixed part and the part moving in the regime of ideal displacement.

In cross flow, the mixing also reduces the efficiency of mass exchange. Unlike the countercurrent flow, it is substantial in the directions of both flows. However, this feature of mixing in cross flow can also be evaluated by the amount φ of a totally mixed liquid and the amount 1 – φ of an ideally displaced liquid.

In total mixing of the entire liquid on a plate, the vapor leaving the plate is in equilibrium with the discharging liquid:

$$(y_n^*)_{\rm mix} = mx_{n-1}^* .$$

In ideal displacement, the vapor leaving the initial portion of an ideal plate is in equilibrium with the arriving liquid (Fig. 1):

$$(y_{n,\rm in}^*)_{\rm dis} = mx_n^* ,$$

while the vapor leaving the final portion is in equilibrium with the leaving liquid:

$$(y_{n,\rm fin}^*)_{\rm dis} = mx_{n-1}^* .$$

The average composition of the vapor after the plate of ideal displacement on condition that the depletion of the liquid is uniform is

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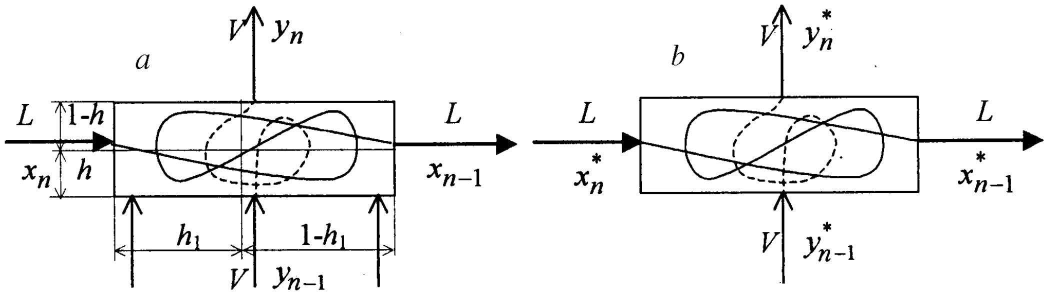

Fig. 1. Scheme of the cross motion of the vapor and liquid phases on real (a) and ideal (b) plates in mixing.

$$(y_n^*)_{\text{dis}} = \frac{(y_{n,\text{in}}^*)_{\text{dis}} + (y_{n,\text{fin}}^*)_{\text{dis}}}{2} = m\,\frac{x_n^* - x_{n-1}^*}{2}\,.$$

With allowance for the adopted evaluation of mixing, the composition of the vapor after an ideal plate is

$$y_n^* = \varphi\,(y_n^*)_{\text{mix}} + (1-\varphi)\,(y_n^*)_{\text{dis}} = \varphi m x_{n-1}^* + (1-\varphi)\,m\,\frac{x_n^* - x_{n-1}^*}{2}\,. \tag{1}$$

By analogy with the cross motion of the phases without mixing [2], we can use the following model of mass exchange in the presence of mixing. The arriving and outgoing flows of a vapor and a liquid on ideal and real plates differ in composition and are equated at a certain distance h for the vapor and h_1 for the liquid from the site of injection of the flows. The equations of equality of the concentrations of a highly volatile component in the vapor phase and the liquid are analogous to the corresponding equations of cross flow without mixing [2]:

$$hy_n^* + (1-h)\,y_{n-1}^* = hy_n + (1-h)\,y_{n-1}\,, \tag{2}$$

$$(1-h_1)\,x_n^* + h_1 x_{n-1}^* = (1-h_1)\,x_n + h_1 x_{n-1}\,. \tag{3}$$

The equations of material balance of the highly volatile component for ideal and real plates and the expressions of the efficiency in the vapor and liquid phases have the same form as in concurrent motion [3].

The concentrations of the highly volatile component in the vapor phase before and after an ideal plate expressed in terms of the parameters of an actual plate retain their form in the absence of mixing as well [2]

$$y_{n-1}^* = hy_n + (1-h)\,y_{n-1} - h\,\frac{y_n - y_{n-1}}{E_{\text{v}}}\,, \tag{4}$$

$$y_n^* = hy_n + (1-h)\,y_{n-1} + (1-h)\,\frac{y_n - y_{n-1}}{E_{\text{v}}}\,, \tag{5}$$

and their difference in the liquid with account for (1) is modified:

$$x_n^* - x_{n-1}^* = \frac{2\,(1-h_1)}{1-2h_1+\varphi}\,x_n + \frac{2h_1}{1-2h_1+\varphi}\,x_{n-1} - \frac{2h}{1-2h_1+\varphi}\,\frac{y_n}{m} -$$

$$-\frac{2(1-h)}{1-2h_1+\varphi}\frac{y_{n-1}}{m}-\frac{2(1-h)}{1-2h_1+\varphi}\frac{y_n-y_{n-1}}{mE_{\text{v}}}\,. \tag{6}$$

Using (4)–(6) to solve the equations of material balance of ideal and real plates [3], we obtain

$$x_n-x_{n-1}=\left(x_{n-1}-\frac{y_{n-1}}{m}\right)E_{\text{v}}\bigg/\left[L\,(1-E_{\text{v}})\left(\frac{1-h}{mV}+\frac{1-h_1}{L}\right)+\frac{L}{mV}E_{\text{v}}-\frac{1}{2}+\frac{\varphi}{2}\right]. \tag{7}$$

The analogous dependence can be found using the efficiency in the liquid, which confirms the equality of the efficiencies in the vapor and liquid phases:

$$E_{\text{v}}=E_{\text{liq}}=E_{\text{k},\varphi}\,. \tag{8}$$

In the particular cases where $h = h_1$ or $h = h_1 = 0.5$, formula (2) takes the corresponding forms:

$$x_n-x_{n-1}=\left(x_{n-1}-\frac{y_{n-1}}{m}\right)E_{\text{k},\varphi}\bigg/\left[(1-E_{\text{k},\varphi})\,(1-h)\left(\frac{L}{mV}+1\right)+\frac{L}{mV}E_{\text{k},\varphi}-\frac{1}{2}+\frac{\varphi}{2}\right], \tag{9}$$

$$x_n-x_{n-1}=\left(x_{n-1}-\frac{y_{n-1}}{m}\right)2E_{\text{k},\varphi,\text{m}}\bigg/\left(\frac{L}{mV}+\frac{L}{mV}E_{\text{k},\varphi,\text{m}}-E_{\text{k},\varphi,\text{m}}+\varphi\right). \tag{10}$$

The boundary conditions of a generalized model are the conditions of the relationship of ideal and real plates inherent in the Murphree [4–6] and Hausen models [5–7] where the distances h and h_1 become equal to zero or to unity. Formula (7) in this case is simplified to the corresponding form:

for $y^*_{n-1}=y_{n-1},\ x^*_{n-1}=x_{n-1}$ ($h=0,\ h_1=1$)

$$x_n-x_{n-1}=\left(x_{n-1}-\frac{y_{n-1}}{m}\right)E_{\text{k},\varphi1}\bigg/\left(\frac{L}{mV}-\frac{1}{2}+\frac{\varphi}{2}\right); \tag{11}$$

for $y^*_n=y_n,\ x^*_n=x_n$ ($h=1,\ h_1=0$)

$$x_n-x_{n-1}=\left(x_{n-1}-\frac{y_{n-1}}{m}\right)E_{\text{k},\varphi2}\bigg/\left(\frac{L}{mV}E_{\text{k},\varphi2}-E_{\text{k},\varphi2}+\frac{1}{2}+\frac{\varphi}{2}\right); \tag{12}$$

for $y^*_{n-1}=y_{n-1},\ x^*_n=x_n$ ($h=0,\ h_1=0$)

$$x_n-x_{n-1}=\left(x_{n-1}-\frac{y_{n-1}}{m}\right)E_{\text{k},\varphi3}\bigg/\left(\frac{L}{mV}-E_{\text{k},\varphi3}+\frac{1}{2}+\frac{\varphi}{2}\right); \tag{13}$$

for $y^*_n=y_n,\ x^*_{n-1}=x_{n-1}$ ($h=1,\ h_1=1$)

$$x_n-x_{n-1}=\left(x_{n-1}-\frac{y_{n-1}}{m}\right)E_{\text{k},\varphi4}\bigg/\left(\frac{L}{mV}E_{\text{k},\varphi4}-\frac{1}{2}+\frac{\varphi}{2}\right). \tag{14}$$

By equating the right-hand sides of dependences (10)–(14) we obtain the relation between the efficiencies of the considered variants:

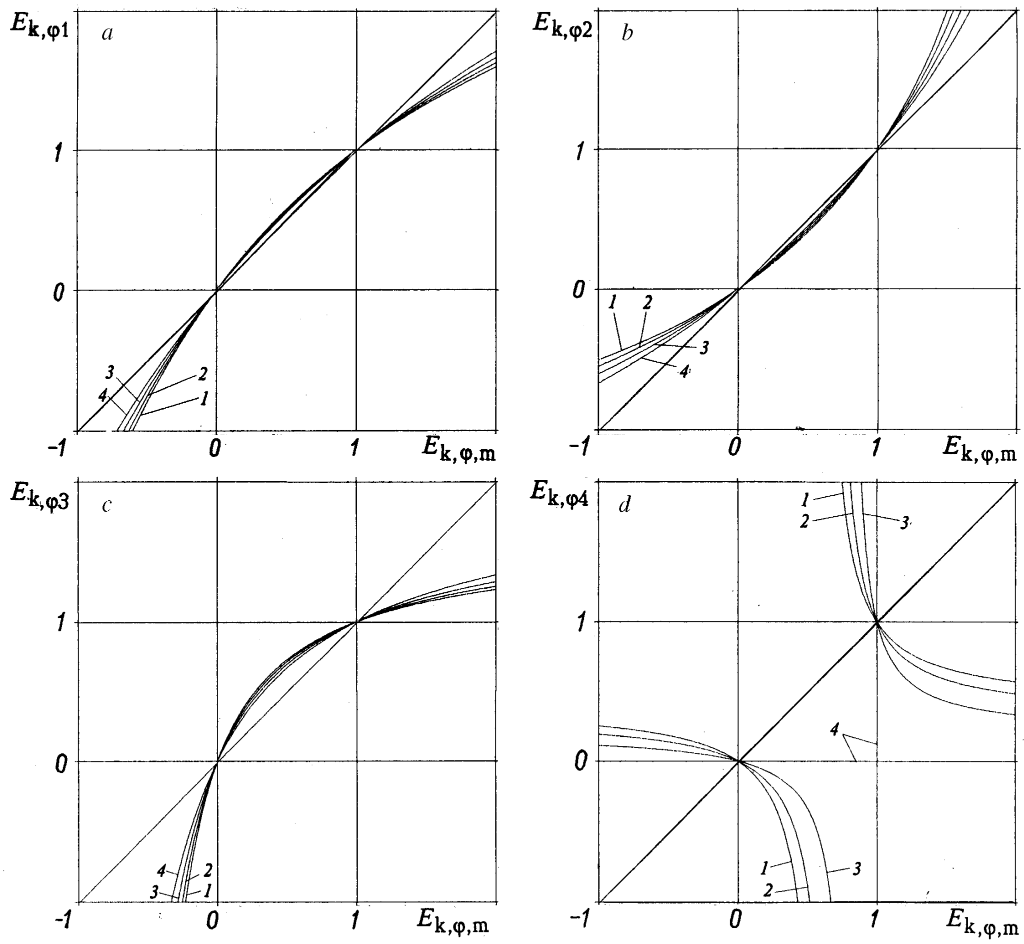

Fig. 2. Dependence of $E_{k,\varphi1}$ (a), $E_{k,\varphi2}$ (b), $E_{k,\varphi3}$ (c), and $E_{k,\varphi4}$ (d) on $E_{k,\varphi,m}$ for L/mV = 1.5 and different values of φ: 1) 0.0; 2) 0.2; 3) 0.5; 4) 1.0.

$$\frac{\frac{L}{mV}+\varphi}{2E_{k,\varphi,m}}+\frac{L}{2mV}-\frac{1}{2}=\frac{\frac{L}{mV}-\frac{1}{2}+\frac{\varphi}{2}}{E_{k,\varphi1}}=\frac{L}{mV}-1+\frac{1+\varphi}{2E_{k,\varphi2}}=\frac{\frac{L}{mV}+\frac{1}{2}+\frac{\varphi}{2}}{E_{k,\varphi3}}-1=\frac{L}{mV}-\frac{1-\varphi}{2E_{k,\varphi4}}\,. \tag{15}$$

We can obtain from (7) and (10) the expressions of the efficiency of a real plate with the known experimental data

$$E_{k,\varphi}=\left[(1-h)\frac{L}{mV}-h_1+\frac{1}{2}+\frac{\varphi}{2}\right]\Bigg/\left(\frac{x_n-\frac{y_{n-1}}{m}}{x_n-x_{n-1}}-h\frac{L}{mV}-h_1\right), \tag{16}$$

$$E_{k,\varphi,m}=\left(\frac{L}{mV}+\varphi\right)\Bigg/\left(\frac{x_n+x_{n-1}-2\frac{y_{n-1}}{m}}{x_n-x_{n-1}}-\frac{L}{mV}\right). \tag{17}$$

As is seen from Fig. 2, mixing in the case of cross flow and L/mV = 1.5 has a small effect on the interrelation of the efficiencies. The efficiency $E_{k,\varphi,m}$ corresponding to the complex model is somewhat lower

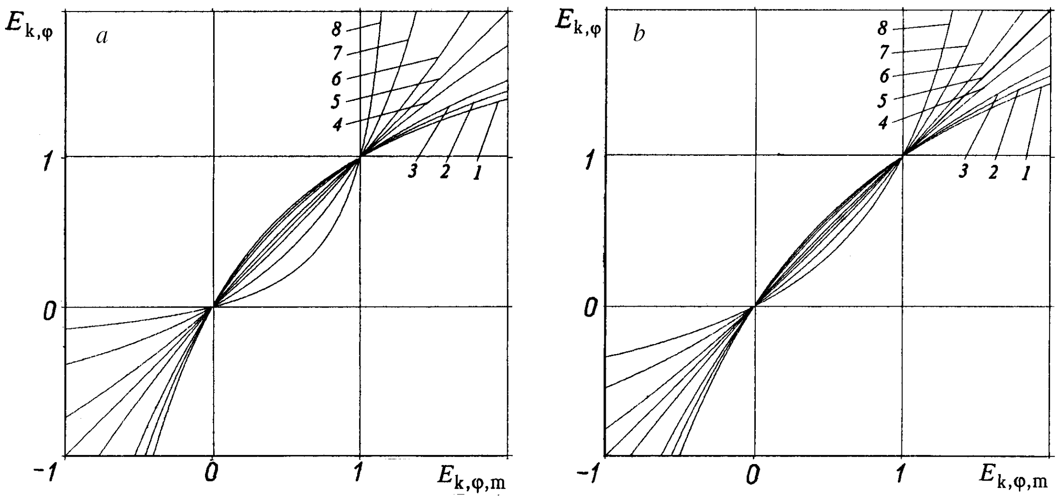

Fig. 3. Dependence of $E_{k,\varphi}$ on $E_{k,\varphi,m}$ for L/mV = 1.5, h_1 = 0.5 (a), and h = 0.5 (b), and also for different values of h (a) and h_1 (b): 1) 0.0; 2) 0.1; 3) 0.2; 4) 0.4; 5) 0.5; 6) 0.6; 7) 0.8; 8) 1.0.

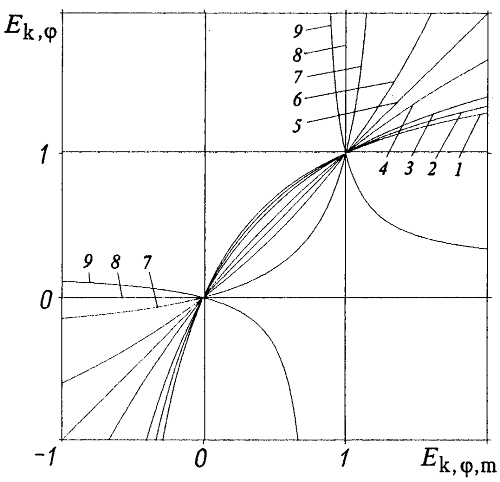

Fig. 4. Dependence of $E_{k,\varphi}$ on $E_{k,\varphi,m}$ for L/mV = 1.5, h_1 = h, and different values of h: 1) 0.0; 2) 0.1; 3) 0.2; 4) 0.4; 5) 0.5; 6) 0.6; 7) 0.8; 8) 0.9; 9) 1.0

than $E_{k,\varphi 1}$, which takes into account the conditions of the relationship of ideal and real plates in the Murphree model in analysis of the efficiency in the vapor phase (Fig. 2a), slightly higher than $E_{k,\varphi 2}$, which reflects the Murphree model in its analysis in the liquid (Fig. 2b), and appreciably lower than $E_{k,\varphi 3}$, which characterizes the Hausen model (Fig. 2c). Mixing somewhat reduces the efficiency in the first and third cases, improves it in the second case, and decreases the difference in the indicated pairs of efficiencies without eliminating it fully in all three cases.

Just as in the absence of mixing in cross flow [2], its presence yields no values of the relations of $E_{k,\varphi,m}$ and $E_{k,\varphi 4}$ in the real range (Fig. 2d). The second of the indicated efficiencies can be considered only in theoretical terms, since the real values of one efficiency correspond to the unreal values of the other. Therefore, the use of $E_{k,\varphi 4}$ in calculations is of limited character and is possible only in the case of unreal values of $E_{k,\varphi,m}$.

When the value of one distance is fixed, a decrease in h (Fig. 3a) or h_1 (Fig. 3b) and also a simultaneous decrease in them (Fig. 4) improves the efficiency. In the latter case, the real efficiencies are possible when $h = h_1 < 0.9$, which is in good agreement with the data of Fig. 2d, for which, as has been noted above, $h = h_1 = 1$.

NOTATION

E, efficiency of the plate; φ, amount of the totally mixed liquid on the plate; h and h_1, dimensionless distance from the site of injection of the vapor and the liquid, respectively, to the surface of equality of the concentrations of the phases on the ideal and real plates; L, molar flow of the liquid; m, coefficient of equilibrium; V, molar flow of the vapor; x and y, concentration of the highly volatile component in the liquid and the vapor, respectively. Subscripts: dis, ideal displacement; φ, allowance for the mixing of the liquid; fin, final portion; k, cross motion of the phases; k,φ, m, cross motion of the phases in mixing and for the values of h and h_1 equal to half their total value; liq, liquid phase; in, initial portion; n, number of the considered plate; n–1, number of the preceding plate in the direction of vapor motion; mix, ideal mixing; v, vapor phase. Superscript: *, ideal conditions.

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